Surface Reconstruction with Triangular Patches from Multiscale Range Images

Yoshifumi KITAMURA, Takeo KIMURA, Masatoshi OKUTOMI and Hideyuki TAMURA
Information Systems Research Center
Canon Inc.
890-12, Kashimada, Saiwai-ku, Kawasaki, 211, Japan
kitamura@cis.canon.co.jp

Abstract

We propose a method to generate shape representation by using various sizes of triangular patches from a range image depending on the surface curvatures of an object. We first generate multiscale range images and then detect edge elements of triangular patches by finding a rapid change of the surface normal in each scale of the image. By integrating these edge elements from multiscale range images successively, sufficient and accurate shape representation can be generated with a small number of triangular patches to represent the three-dimensional object which has a variety of curved and planar surfaces.

1 Introduction

Recent progress of three-dimensional data acquisition devices makes it possible to input accurate shape information of real objects into a computer system. However, the range images obtained by them consist of a large set of data points. Consequently, they are not easy to handle or manipulate for applications such as geometric modelers for computer graphics or computer aided industrial design. Therefore, it is important to reduce the size of the data, while keeping the details of the shape information.

Some research efforts have been directed for this subject [1][2][3][4]. However, since the data sizes of their shape representation depend on the intervals of the grid or the number of the triangles determined in advance, their reductions of the data sizes are not sufficient.

One of the most popular surface models for computer graphics is the shape representation by triangular patches. In order to make the sufficient shape representation by a reasonably small number of triangular patches, it is necessary to generate small patches for surfaces with high curvatures and large patches for those with low curvatures.

We propose a method that meets this requirement, i.e., shape representation by using various sizes of triangular patches depending on the surface curvatures of an object. We first generate multiscale range images and then detect edge elements of triangular patches by finding a rapid change of the surface normal in each scale of the images. By integrating these edge elements from multiscale range images successively, sufficient and accurate shape representation can be generated with a small number of triangular patches to represent the three-dimensional object which has a variety of curved and planar surfaces.

2 Multiscale range images

In this section, we explain the usage of multiscale range images to generate shape representation with triangular patches.

2.1 Triangular patches based on the edge elements

Shape representation with triangular patches consists of three dimensional coordinates of vertexes and sides of each triangle. In order to make the sufficient shape representation by a reasonably small number of triangular patches, it is necessary to generate small patches for surfaces with high curvatures and large patches for those with low curvatures. Therefore, the edges of triangles must be located where the directions of surface normal vectors change rapidly or on contours of the object. We produce triangular patches from these edge elements detected in the range image.

2.2 Detection of edge elements in multiscale range images

Suppose Figure 1(a) shows a cross section of an object surface. (b) and (c) are parts of 1,2 marked in (a) respectively. Three-dimensional objects usually have variety of curved surfaces, e.g., the surface in (b) has larger curvature than (c). In order to generate smaller patches for surfaces with larger curvatures in (b), it is necessary to detect edge elements in a higher scale range image with a smaller pixel interval distance $s_1$. On the contrary, in order to generate larger patches for surfaces with smaller curvatures in (c), it is preferable to detect edge elements in a lower scale range image with a larger pixel interval distance $s_2$. Thus, we generate the shape representation with triangular patches by integrating edge elements from multiscale range images depending on the surface curvatures of an object.

2.3 Relation among resolution, curvature and surface normals

The relation among resolution of the range image, curvature of the surface, and directions of surface normals is examined analytically using a two-dimensional curve. Figure 2 shows a curve which is a cross section of an object surface. Here, this two dimensional curve is analyzed instead of
three-dimensional surfaces for simple explanation. Suppose the surface normal vector at point P on this curve \( z = f(x) \) is \( \vec{n} = \left( -\frac{dz}{dx}, 1 \right) \), and \( \phi \) is the angle formed by x-axis and \( \vec{n} \), the gradient \( \frac{dz}{dx} \) is given by:

\[
\frac{d\phi}{dx} = \frac{\frac{dz}{dx}}{1 + \left( \frac{dz}{dx} \right)^2}
\]

Let \( w(x) \) be the magnitude of \( \vec{n} \) at point P. The angle with two surface normal vectors which are located with the distance \( w(x) \) is approximated as:

\[
\theta(x) \approx \left| \frac{d\phi}{dx} \right| w(x)
\]

On the other hand, the curvature \( \kappa(x) \) at P is given by:

\[
\kappa(x) = \frac{\frac{d^2z}{dx^2}}{\left(1 + \left( \frac{dz}{dx} \right)^2 \right)^{3/2}}
\]

Therefore, from Eq.(1)/(2)/(3), \( w(x) \) is given by:

\[
w(x) = \frac{\theta(x)}{\kappa(x)} \sqrt{1 + \left( \frac{dz}{dx} \right)^2}
\]

In short, the distance of two pixels which produce the angle with two surface normal vectors \( \theta(x) \) is given by \( w(x) \) in Eq.(3) depending on the curvature \( \kappa(x) \) and gradient \( \frac{dz}{dx} \).

### 2.4 Multiscale range image and edge detection

Suppose the distance between two sample points \( w(x) = w_0 \) and angle with two surface normal vectors \( \theta(x) = \theta_1 \) are constant in Eq.(4), the Eq.(5) is obtained as a solution which satisfies Eq.(4).

\[
y = f(x) = -k \cdot \log\cos(x/k) + C
\]

where, \( k = \frac{\pi}{w_0} \) and C are constant. Figure 3(a) shows a curve which consists of three curves of Eq.(5). Since each interval \( (A, B, C) \) has a constant value of \( w_0 = w_1 < w_2 = w_0 < w_3 \) in Eq.(5) respectively, the angle with two surface normal vectors \( \vec{n} \) of each point on the surface are calculated by convoluting the partial differentiation of

---

596
G(x, y) to range image Z(x, y):

\[
\tilde{r}(x, y) = \frac{1}{\sqrt{1 + \left(\frac{\partial G}{\partial x} + Z\right)^2 + \left(\frac{\partial G}{\partial y} + Z\right)^2}} \left(\frac{\partial G}{\partial x} + Z, \frac{\partial G}{\partial y} + Z, -1\right)
\]

(7)

where, \(T/\sigma = 4s\) is to remove larger curvature components than that given by the pixel interval \(w_0 = 2s, s\) is distance between the pixels, and \(T\) is the diameter of Gaussian window.

**Generation of edge map:** Jump edges and roof edges are detected in each scale image. Only the detection of the latter is described here. If the angle with two surface normal vectors at a distance of 2s is greater than threshold \(\theta\), the edge element is detected between these two points. The edge map is generated as the result of the logical sum of these two types of edge elements for each pixel.

**Integration of edge maps:** The edge map of a lower scale is integrated to edge map of a higher scale successively. Closed edges are obtained by removing isolated edge elements and extending broken edge elements in a lower scale. Then they are integrated into edge maps in a higher scale successively.

**Generation of triangular patches:** After the polygons formed by the edge elements generated above are divided into triangles, a shape representation with triangular patches are generated from the coordinates of vertexes, sides of each triangle, and surface normals of the vertexes.

4 Experimental results

This section presents the experimental results using two kinds of range images. The image size is 256×256 in the experiment below.

4.1 Surface with single curvature component and flat surface

Figure 5(a) shows the range image of pentaprism of a single-lens reflex camera obtained by our range finder. This object consists of planar surfaces, however, intersecting lines of these planes are rounded with an almost same curvature. Thus shape representation with triangular patches has been generated from a single-scale range image (s=4). (b) shows the detected edge map, where \(T=256, \theta=10\text{(deg)}\). This object has been represented by 2216 triangular patches.

4.2 Surface with different curvature components

A range image obtained by a range finder[5] has also been used for our experiment. Figure 6(a) shows a range image of ‘mask’. Since this object consists of several curvature components, range images in four scales (s=16,8,4,2) are used. (b) shows the surface normal vectors of the range image smoothed by \(s=16, T=256\). Figure 7(a) shows the edge map generated by \(s=16, \theta=10\text{(deg)}\). (b)(c), and (d) show the edge maps integrated by \(s=8,4,2\) into \(s=16\) respectively. Figure 8 shows a rendered image of the shape representation with triangular patches generated from (d).

5 Discussion

We compare the experimental results with those obtained by using triangular patches which are generated by nodes of a regular grid as the vertexes of triangles. In case of the pentaprism image, the number of triangular patches generated from the regular grid whose interval is 4 (same condition as the experiment in 4.1) is 7938. Therefore, our proposed method can reduce the data size to 27.9\%.
6 Conclusions

In this paper, we have proposed a method to generate the shape representation with various sizes of triangular patches depending on the surface curvatures of an object. We first generate multiscale range images and then detect edge elements of triangular patches by finding a rapid change of the surface normal in each scale of the image. By integrating these edge elements from multiscale range images successively, sufficient and accurate shape representation could be generated with a small number of triangular patches to represent the three-dimensional object which has a variety of curved and planar surfaces.

Acknowledgement: The authors wish to thank Mr. Yoshizaki and Mr. Shimizu of Information Systems Research Center, Canon Inc. for encouragement on this research, and Mr. Tachikawa of Production Engineering Research Lab for his assistance with data acquisition.

References


